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PHYSICAL MODEL OF FLOW WITH ANISOTROPIC LAYER IMPREGNATION

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Isothermal filling of a plane cavity by a Newtonian fluid with simultaneous anisotropic impregnation of a reinforcing layer is considered.

The study of the process of filling of a plane channel (or cavity) with permeable walls by a fluid is of great practical and scientific interest. In nature, this situation can occur in the filling cracks of permeable rocks by fluid. As for engineering practice, in recent years the process of forming different tars or plastisols has found wider use for producing reinforced products. In this case filling of a forming cavity is accompanied by simultaneous impregnation of a reinforcing layer. Low viscosity of the material allows one to rather quickly perform the process of product forming with complete impregnation of all porous elements. At the next stage the casting solidifies due to material cross-linking during the gelling process [1].

At present a number of works in the literature are devoted to the formation of composites in closed moulds [2-8]. However, these publications consider injection to form cavities the entire inner volume of which is occupied with some filler. Therefore, cavity filling resides only in the impregnation of a porous material. A mathematical model of this process is based, as a rule, on the Darci law [3-8] describing fluid flow through a permeable material.

On forming polymer products with a reinforcing core, a shear flow in the cavity is associated with simultaneous impregnation of a porous layer (Fig. 1). A similar problem of low-pressure cast formation of plastisols with impregnation of a fabric substrate was solved in [9]. Assuming the fabric layer to be very thin the authors of [9] started from an approximation of "rapid" impregnation

$$l_x(t) >> l_x(t) - x_0(t),$$

where x_0 is the length of the completely impregnated fabric; l_x is the length of the impregnation zone; t is the time.

Thus, it was assumed that impregnation does not occur along the entire length l, but in a narrow zone adjacent to the flow front in the cavity. Fabric impregnation was considered to be one-dimensional and the pressure profile over the fabric depth to be linear. As a result of the solution it was found that the rate of impregnation is independent of the axial coordinate, the moving boundary is linear, and the length of the real zone of impregnation $l_x - x_0$ does not change during filling. The solution is reduced to compact formulas for calculating the time of cavity filling and the time necessary for complete impregnation of the fabric layer.

In the present paper, a solution of the problem of formation with reinforcing layer impregnation in a most complete formulation embracing a wide range of possible cases is obtained.

An isothermal process of low-pressure cast formation of reinforced plastisol products is considered on the basis of the scheme shown in Fig. 1, in accordance with which a long plane forming a cavity of height $2h$ and length l is filled through a plane-slot sprue channel under constant pressure p_0. The process of cavity filling is accompanied by simultaneous impregnation of a porous reinforcing layer with thickness H. As the flow front moves, in a plane cavity there takes place two-dimensional propagation of the plastisol through a permeable medium with its own

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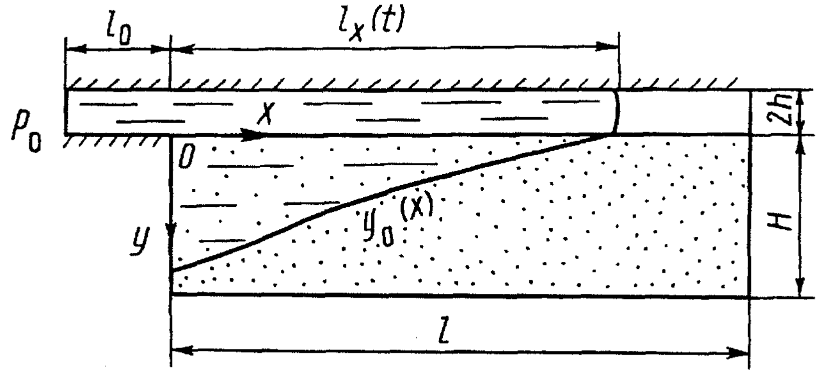

Fig. 1. General scheme of the formation process.

developing front. The whole forming process can be conventionally divided to three stages. The first stage terminates when the flow front in the cavity reaches the opposite wall. The second stage lasts until the reinforcing layer is completely impregnated. The third stage ends when the pressure is leveled through the entire volume of the product.

In modeling the considered process, the plastisol was considered to be an incompressible Newtonian fluid [10]. The fluid flow in a plane cavity obeys the Navier-Stokes equation. Here it should be noted that peculiarities in the distribution of velocities in the flow front region are not considered.

Assuming the pressure in the cavity $p_c = p_c(x)$ to be independent of the transverse coordinate y, we restrict ourselves to one equation of motion

$$\mu \frac{\partial^2 u_c}{\partial y^2} = \frac{dp_c}{dx}, \tag{1}$$

which is supplemented by the continuity equation

$$\frac{\partial u_c}{\partial x} + \frac{\partial v_c}{\partial y} = 0. \tag{2}$$

The boundary conditions for the flow in the cavity are

$$y = -2h: \ u_c = v_c = 0, \tag{3}$$

$$y = 0: \ u_c = u, \quad v_c = v, \tag{4}$$

$$x = l_x(t): \ p_c = 0, \tag{5}$$

$$x = 0: \ \bar{u}_{c0} = \frac{h^2}{3\mu} \frac{p_0 - p_{c0}}{l_0}, \tag{6}$$

where $pc_0 = p_c(x = 0)$, $\bar{u}_{\infty} = \bar{u}_c(x = 0)$.

When the flow front reaches the opposite wall of the cavity, condition (5) is replaced by the following

$$x = l: \ \frac{dp_c}{dx} = 0. \tag{7}$$

Modeling of the impregnation of a reinforcing layer was based on the Darci equation [3-8] under the assumption of possible anisotropy of the impregnated material. This can be caused, for example, by the fact that the reinforcing element of the product can be produced by applying a great number of thin plane layers to each other. In this case, the permeability along these layers k_x differs, as a rule, from the permeability in the perpendicular direction k_y. Thus, for the components of the filtration velocity u and v we have, according to the Darci law,

$$u = -\frac{k_x}{\mu}\frac{\partial p}{\partial x}, \quad v = -\frac{k_y}{\mu}\frac{\partial p}{\partial y}. \tag{8}$$

Substituting (8) into the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{9}$$

we obtain the Laplace equation

$$k_x \frac{\partial^2 p}{\partial x^2} + k_y \frac{\partial^2 p}{\partial y^2} = 0, \tag{10}$$

which describes the field of fluid pressure in a permeable body.

It should be noted that the formulation of the boundary condition (4) $u = u_c$ along with the use of the Darci equation has an approximate character. This is related to the fact that the boundary $y = 0$ separates the shear flow zone in the channel from one side and the potential flow zone in the porous layer from the other. Thus, discontinuity of shear stresses is admitted on a permeable wall. A more correct formulation of the problem is related to the presence of some boundary layer inside the porous body near the boundary $y = 0$, which represents the transition zone between shear and potential flows. In this case the condition of the equality of not only the velocity but shear stresses as well is formulated on a permeable wall. However, this approach in the situation considered could lead, to our mind, to an unjustified complication of the problem.

At the first stage of the process, when the impregnation front in the cavity has not reached the remote wall ($l_x < l$) and the moving impregnation front has not penetrated the full length of a porous layer, the boundary conditions for (10) have the form

$$y = 0 \quad p = p_c(x), \tag{11}$$

$$x = 0 \quad \frac{\partial p}{\partial x} = 0, \tag{12}$$

$$y = y_0(x) \quad p = 0. \tag{13}$$

As the impregnation front reaches one or another opposite wall of the cast mould, boundary conditions (11)-(13) are supplemented by the following conditions (the conditions of wall impermeability)

$$y = H \quad \frac{\partial p}{\partial y} = 0, \tag{14}$$

$$x = l \quad \frac{\partial p}{\partial x} = 0. \tag{15}$$

Integrating (1) twice with allowance for boundary conditions (3) and (4), we find

$$u_c = \frac{1}{\mu}\frac{dp_c}{dx}\left[\frac{y^2}{2} + \left(h - \frac{k_x}{2h}\right)y - k_x\right]. \tag{16}$$

It is not difficult to obtain from (16) an expression for the mean velocity

$$\bar{u}_c = -\frac{k_x}{\mu}\frac{dp_c}{dx}\beta, \tag{17}$$

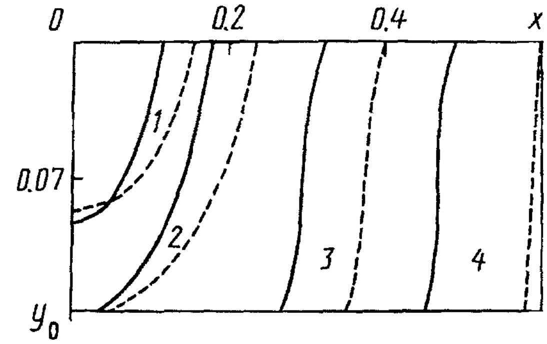

Fig. 2. Development of impregnation front for isotropic (solid lines, $k_x = k_y = 7.4\cdot10^{-8}$) and anisotropic (dash lines, $k_y = 7.4\cdot10^{-8}$, $k_x = 2k_y$) permeabilities of porous body: 1, t = 1.3 sec; 2, 3.0; 3, 7.5; 4, 13.9 sec. y_0, x, m.

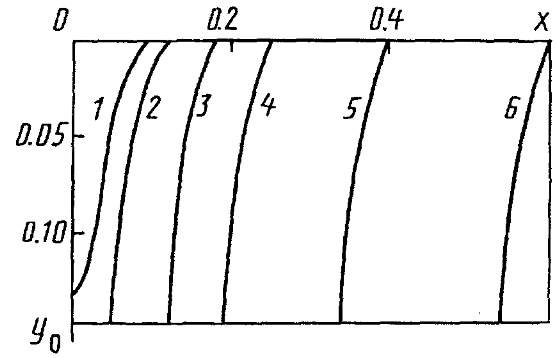

Fig. 3. Development of impregnation front for anisotropic permeability when $k_x = k_y/20$; $k_y = 7.4\cdot10^{-8}$ m^2: 1, t = 1.4 sec; 2, 2.9; 3, 7.5; 4, 14.1; 5, 34.5; 6, t = 76.7 sec.

where $\beta = h^2/(3k_x)+0.5$.

From the continuity equation it is easy to find the velocity component v_c and, in particular, its value at $y = 0$

$$v_c\,(y=0) = -\int_{-2h}^{0}\frac{\partial u_c}{\partial x}\,dy = \frac{2hk_x}{\mu}\,\frac{d^2 p_c}{dx^2}\,\beta\,. \tag{18}$$

Substituting (18) into boundary condition (4) and using equation (8), we find an equation for the pressure distribution along the plane cavity $p_c(x)$:

$$\frac{d^2 p_c}{dx^2} = -\frac{1}{2\,h\,\beta\,\kappa^2}\,\frac{\partial p}{\partial y}\bigg|_{y=0}\,, \tag{19}$$

where $\kappa = \sqrt{k_x/k_y}$.

Thus, a mathematical model of the considered process is expressed by system of differential Eqs. (19) and (10), the solution of which with the given boundary conditions allows one to find the pressure fields in the cavity and in a permeable layer for each time instant. In spite of the nonstationary character of the process, time enters into all the determining relations as a parameter. Time affects pressure and velocity fields in terms of the variable boundaries of the flow region. Thus, for the points of a moving front of impregnation in a porous body we have

$$\frac{dy_0}{dt} = \frac{1}{\varepsilon}\,v\,, \quad \frac{dx_0}{dt} = \frac{1}{\varepsilon}\,u\,, \tag{20}$$

where x_0, y_0 are the coordinates of the impregnation front.

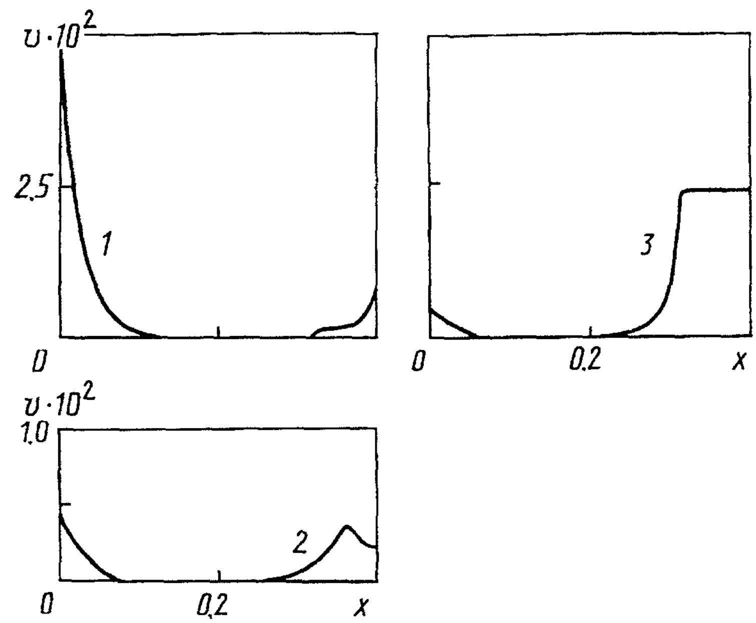

Fig. 4. Distribution of vertical component of filtration velocity along permeable channel wall ($y = 0$): 1, $t = 10.6$ sec, $k_{x,D} - k_y = 7.4 \cdot 10^{-8}$ m^2; 2, $t = 34.5$ sec, $k_x = 7.4 \cdot 10^{-8}$ m^2, $k_x = k_y/20$; 3, $t = 0.2$ sec, $k_x = k_y = 10^{-7}$ m. $v \cdot 10^2$, m/sec.

The position of the impregnation front in the cavity is determined by the relation

$$\frac{dl_x}{dt} = \bar{u}_c . \tag{21}$$

A combined solution of the system was performed by an iteration scheme at each step of which Eq. (10) was solved by the Seidel method and Eq. (19) by the factorization method [11].

Concrete calculations illustrating specific features of the solution were performed for $p_0 = 10^5$ Pa; $\varepsilon = 0.5$; $\mu = 3$ Pa·sec; $2h = 0.01$ m.

The calculations of the anisotropic impregnation showed that at

$$\alpha = \frac{l}{2h\beta} \leq 1$$

the change in the permeability k_x at constant k_y has practically no affect on the calculation results. Another situation arises when $\alpha >> 1$. It is seen from Fig. 2 that at $\alpha = 100$ the two-fold variation of the permeability k_x considerably changes the profile of the impregnation front and the total time of product formation.

Another version of anisotropic impregnation, when k_x is much smaller than k_y, is given in Fig. 3. Comparison of Figs. 2 and 3 shows that reduction of axial permeability k_x leads to both substantial retardation of the process of porous body impregnation and propagation of the flow front in a plane cavity. Simultaneously, a three- or four-fold decrease in volumetric flow rate of fluid through the inlet section of the cavity occurs ($x = 0$). The reason for this phenomenon is well illustrated in Fig. 4. Here, the distribution of the filtration velocity v over the surface of a porous channel wall ($y = 0$) that corresponds to different time instants but the same length of fluid flowing into the cavity ($l_x = 0.4$ m) is shown for different cases of formation. Curve 1 reflects the case of isotropic impregnation corresponding to Fig. 2. This distribution of $v(x)$ indicates the fact that the main volume of fluid gets into the porous body in the section $0 < x < 0.1$ m and then, by streamlines, it moves along the porous layer. Thus, impregnation is performed here mainly at the expense of the horizontal fluid flow inside the reinforcing layer. Curve 2 corresponds to the calculation version presented in Fig. 3. A sharp reduction of k_x greatly decreases the transmissive capacity of the porous body in a horizontal direction, and the impregnation front is to a great extent determined by the fluid fed to the region $l_x(t) - x_0(t)$. This is the cause of a noticeable reduction of the velocity of flow front propagation along the plane cavity.

Figures 2 and 3 show that the fulfillment of the condition $l_x(t)-x_0(t) << l_x(t)$ is still not sufficient to consider impregnation as one-dimensional. The following quantity

$$\xi = \frac{l}{\alpha H},$$

which reflects the ratio of fluid flow rates inside the porous body in the vertical and axial directions, respectively, can be used as an applicability criterion of the theory of one-dimensional impregnation. In this case, when $\xi >> 1$, horizontal fluid flow through the porous layer can be neglected. To curve 1 in Fig. 4 there corresponds $\xi = 0.04$, and to curve 2, $\xi = 0.73$. Here the model of one-dimensional impregnation cannot be applied. Curve 3 in Fig. 4 is an example of the situation when the horizontal filtration velocity can be neglected (in spite of the fact that $k_x = k_y$). In this case $H = 0.02$ m and $\xi = 25$. If impregnation is really considered one-dimensional, then the conclusion of [9] that the filtration velocity v in the region l_x-x_0 is independent of the axial coordinate is confirmed.

NOTATION

p, p_c, fluid pressure in porous medium and in cavity, respectively; u, v, longitudinal and transverse components of filtration velocity; u_c, v_c, longitudinal and transverse components of flow velocity in cavity; $\overline{u}_c$, mean velocity in cavity; h, half-height of cavity; H, porous layer height; l_0, l, length of sprue channel and cast mould, respectively; μ, fluid viscosity; $y_0(t)$, coordinate of the impregnation front; $x_0(t)$, axial coordinate at which $y_0 = H$; ε, impregnated material porosity; k_x, k_y, longitudinal and transverse coefficients of permeability. Dimensionless quantities and parameters: $\alpha = l/2h\beta$; $\beta = h^2/(3k_x)+0.5$; $\kappa = \sqrt{k_x/k_y}$.

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